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THE PRESERVATION THEOREMS OF FUZZY (r, s)-SEMI-IRRESOLUTE MAPPINGS

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Abstract. In this paper, we prove that (r, s)-semi-irresolute image of the (r, s)-semi-θ-connected set is also (r, s)-semi-θ-connected. Moreover, we prove that an (r, s)-semi-irresolute mapping preserves (r, s)-S^*-closedness.

1. Introduction

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Çoker [5] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [6] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth topological spaces and intuitionistic fuzzy topological spaces. S. Malakar [11] introduced the concept of fuzzy semi-irresolute mappings, and S. H. Cho and J. K. Park [4] established some other properties of fuzzy semi-irresolute mappings on Chang's fuzzy topological spaces. S. J. Lee and J. T. Kim [8] introduced the concept of fuzzy (r, s)-semi-irresolute mappings, and investigated some of their characteristic properties.

In this paper, we prove that (r, s)-semi-irresolute image of the (r, s)-semi-θ-connected set is also (r, s)-semi-θ-connected. Moreover, we prove that an (r, s)-semi-irresolute mapping preserves (r, s)-S^*-closedness.

2. Preliminaries

For the nonstandard definitions and notations we refer to [7, 9, 10].

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DEFINITION 2.1 ([6]). Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense*(SoIFT for short) $\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2)$ on X is a mapping $\mathcal{T} : I(X) \to I \otimes I$ which satisfies the following properties:

(1) $\mathcal{T}_1(\underline{0}) = \mathcal{T}_1(\underline{1}) = 1$ and $\mathcal{T}_2(\underline{0}) = \mathcal{T}_2(\underline{1}) = 0$.
(2) $\mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \wedge \mathcal{T}_1(B)$ and $\mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B)$.
(3) $\mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i)$ and $\mathcal{T}_2(\bigcup A_i) \leq \bigvee \mathcal{T}_2(A_i)$.

The $(X, \mathcal{T}) = (X, \mathcal{T}_1, \mathcal{T}_2)$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense*(SoIFTS for short). Also, we call $\mathcal{T}_1(A)$ a *gradation of openness* of A and $\mathcal{T}_2(A)$ a *gradation of nonopenness* of A.

DEFINITION 2.2 ([8]). Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is said to be *fuzzy* (r, s)*-semi-irresolute* if for each intuitionistic fuzzy set $x_{(\alpha,\beta)}$ in X and each fuzzy (r, s)-semiopen set B in Y with $f(x_{(\alpha,\beta)}) \in B$, there is a fuzzy (r, s)-semiopen set A in X such that $x_{(\alpha,\beta)} \in A$ and $f(A) \subseteq \mathrm{scl}(B, r, s)$.

LEMMA 2.3 ([8]). *Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then the following statements are equivalent:*

(1) *A is fuzzy (r, s)-regular semiopen.*
(2) *$A = scl(sint(A, r, s), r, s)$.*
(3) *A is fuzzy (r, s)-semi-clopen.*

LEMMA 2.4 ([8]). *Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. If A is a fuzzy (r, s)-semiopen set in X, then $scl(A, r, s)$ is fuzzy (r, s)-regular semiopen in X.*

THEOREM 2.5 ([8]). *Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then the following statements are equivalent:*

(1) *f is fuzzy (r, s)-semi-irresolute.*
(2) *For each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r, s)-semi-θ-clopen set B containing $f(x_{(\alpha,\beta)})$, there is a fuzzy (r, s)-semi-θ-clopen set A such that $x_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$.*
(3) *For each fuzzy (r, s)-regular semiopen set B in Y, $f^{-1}(B)$ is fuzzy (r, s)-regular semiopen in X.*
(4) *For each fuzzy (r, s)-semiopen set B in Y,*
$$f^{-1}(B) \subseteq sint_\theta(f^{-1}(scl_\theta(B, r, s)), r, s).$$
(5) *For each fuzzy (r, s)-semiclosed set B in Y,*
$$scl_\theta(f^{-1}(sint_\theta(B, r, s)), r, s) \subseteq f^{-1}(B).$$

(6) *For each fuzzy (r,s)-semiopen set B in Y,*

$$scl_\theta(f^{-1}(B),r,s) \subseteq f^{-1}(scl_\theta(B,r,s)).$$

3. Main results

DEFINITION 3.1. Let $(X,\mathcal{T})$ be a SoIFTS and $(r,s)\in I\otimes I$. Then X is said to be *fuzzy (r,s)-semi-θ-T_2* if for each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ and $y_{(\gamma,\delta)}$ in X with $x\neq y$, there exist $A\in N_{\mathrm{s}}^{\mathrm{q}}(x_{(\alpha,\beta)})$ and $B\in N_{\mathrm{s}}^{\mathrm{q}}(y_{(\gamma,\delta)})$ such that $\mathrm{scl}(A,r,s)\cap\mathrm{scl}(B,r,s)=\underline{0}$.

THEOREM 3.2. *Let $f:(X,\mathcal{T})\to(Y,\mathcal{U})$ be an injective fuzzy (r,s)-semi-irresolute mapping and $(r,s)\in I\otimes I$. If Y is fuzzy (r,s)-semi-θ-T_2, then X is fuzzy (r,s)-semi-θ-T_2.*

Proof. Let $x_{(\alpha,\beta)}$ and $y_{(\gamma,\delta)}$ be intuitionistic fuzzy points in X with $x\neq y$. Since f is injective, we have $f(x_{(\alpha,\beta)})\neq f(y_{(\gamma,\delta)})$. Because Y is fuzzy (r,s)-semi-θ-T_2, there exist $A\in N_{\mathrm{s}}^{\mathrm{q}}(f(x_{(\alpha,\beta)}))$ and $B\in N_{\mathrm{s}}^{\mathrm{q}}(f(y_{(\gamma,\delta)}))$ such that $\mathrm{scl}(A,r,s)\cap\mathrm{scl}(B,r,s)=\underline{0}$ in Y. Since $\mathrm{scl}(A,r,s)$ and $\mathrm{scl}(B,r,s)$ are fuzzy (r,s)-regular semiopen, by Theorem 2.5, $f^{-1}(\mathrm{scl}(A,r,s))$ and $f^{-1}(\mathrm{scl}(B,r,s))$ are fuzzy (r,s)-regular semiopen in X. Moreover,

$$f^{-1}(\mathrm{scl}(A,r,s))\in N_{\mathrm{s}}^{\mathrm{q}}(x_{(\alpha,\beta)}),\ \ f^{-1}(\mathrm{scl}(B,r,s))\in N_{\mathrm{s}}^{\mathrm{q}}(y_{(\gamma,\delta)})$$

and

$$\mathrm{scl}(f^{-1}(\mathrm{scl}(A,r,s)),r,s)\cap\mathrm{scl}(f^{-1}(\mathrm{scl}(B,r,s)),r,s)=f^{-1}(\underline{0})=\underline{0}.$$

Hence X is fuzzy (r,s)-semi-θ-T_2. □

DEFINITION 3.3. Let $(X,\mathcal{T})$ be a SoIFTS and $(r,s)\in I\otimes I$. Then X is said to be

(1) *fuzzy (r,s)-semi-θ-disconnected* if there exist two fuzzy (r,s)-semiopen sets A_1 and A_2 with $A_1\neq\underline{0}$ and $A_2\neq\underline{0}$ such that

$$\mathrm{scl}(A_1,r,s)\cap\mathrm{scl}(A_2,r,s)=\underline{0}\text{ and }\mathrm{scl}(A_1,r,s)\cup\mathrm{scl}(A_2,r,s)=\underline{1},$$

(2) *fuzzy (r,s)-semi-θ-connected* if it is not fuzzy (r,s)-semi-θ-dis connected.

THEOREM 3.4. *Let $f:(X,\mathcal{T})\to(Y,\mathcal{U})$ be a surjective fuzzy (r,s)-semi-irresolute mapping and $(r,s)\in I\otimes I$. If X is fuzzy (r,s)-semi-θ-connected, then Y is fuzzy (r,s)-semi-θ-connected.*

Proof. Suppose that Y is not fuzzy (r,s)-semi-θ-connected. Then there are two fuzzy semiopen sets B_1 and B_2 in Y with $B_1 \neq \underline{0}$ and $B_2 \neq \underline{0}$ such that

$$\mathrm{scl}(B_1,r,s)\cap \mathrm{scl}(B_2,r,s)=\underline{0} \text{ and } \mathrm{scl}(B_1,r,s)\cup \mathrm{scl}(B_2,r,s)=\underline{1}.$$

Since $\mathrm{scl}(B_1,r,s)\neq \underline{0}$ and $\mathrm{scl}(B_2,r,s)\neq \underline{0}$, we obtain

$$f^{-1}(\mathrm{scl}(B_1,r,s))\neq \underline{0} \text{ and } f^{-1}(\mathrm{scl}(B_2,r,s))\neq \underline{0}.$$

Because $\mathrm{scl}(B_1,r,s)$ and $\mathrm{scl}(B_2,r,s)$ are fuzzy (r,s)-regular semiopen in Y, $f^{-1}(\mathrm{scl}(B_1,r,s))$ and $f^{-1}(\mathrm{scl}(B_2,r,s))$ are fuzzy (r,s)-regular semiopen in X. Moreover,

$$\begin{aligned}&\mathrm{scl}(f^{-1}(\mathrm{scl}(B_1,r,s)),r,s)\cap \mathrm{scl}(f^{-1}(\mathrm{scl}(B_2,r,s)),r,s)\\ &\quad = f^{-1}(\mathrm{scl}(B_1,r,s)\cap \mathrm{scl}(B_2,r,s))=\underline{0}\end{aligned}$$

and

$$\begin{aligned}&\mathrm{scl}(f^{-1}(\mathrm{scl}(B_1,r,s)),r,s)\cup \mathrm{scl}(f^{-1}(\mathrm{scl}(B_2,r,s)),r,s)\\ &\quad = f^{-1}(\mathrm{scl}(B_1,r,s)\cup \mathrm{scl}(B_2,r,s))=\underline{1}.\end{aligned}$$

Therefore X is not fuzzy (r,s)-semi-θ-connected. □

DEFINITION 3.5. Let $(X,\mathcal{T})$ be a SoIFTS and $(r,s)\in I\otimes I$. Then X is said to be *fuzzy* (r,s)-S^*-*closed* if for each fuzzy (r,s)-semiopen cover $\{B_\alpha \mid \alpha\in\Lambda\}$ of X, there is a finite subset Λ_0 of Λ such that $\bigcup_{\alpha\in\Lambda_0}\mathrm{scl}(B_\alpha,r,s)=\underline{1}$.

THEOREM 3.6. *Let $f:(X,\mathcal{T})\to(Y,\mathcal{U})$ be a surjective fuzzy (r,s)-semi-irresolute mapping and $(r,s)\in I\otimes I$. If X is fuzzy (r,s)-S^*-closed, then Y is fuzzy (r,s)-S^*-closed.*

Proof. Let $\{B_\alpha \mid \alpha\in\Lambda\}$ be a fuzzy (r,s)-semiopen cover of Y. Then $\{\mathrm{scl}(B_\alpha,r,s)\mid \alpha\in\Lambda\}$ is a fuzzy (r,s)-regular semiopen cover of Y. By Theorem 2.5, $\{f^{-1}(\mathrm{scl}(B_\alpha,r,s))\mid\alpha\in\Lambda\}$ is a fuzzy (r,s)-regular semiopen cover of X. Since X is fuzzy (r,s)-S^*-closed, there is a finite subset Λ_0 of Λ such that

$$\bigcup_{\alpha\in\Lambda_0} f^{-1}(\mathrm{scl}(B_\alpha,r,s))=\bigcup_{\alpha\in\Lambda_0}\mathrm{scl}(f^{-1}(\mathrm{scl}(B_\alpha,r,s)),r,s)=\underline{1}.$$

Because f is surjective, we obtain

$$\underline{1}=f(\underline{1})=f(\bigcup_{\alpha\in\Lambda_0} f^{-1}(\mathrm{scl}(B_\alpha,r,s)))=\bigcup_{\alpha\in\Lambda_0}\mathrm{scl}(B_\alpha,r,s).$$

Hence Y is fuzzy (r,s)-S^*-closed. □

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